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Abstract. *In the present work, we are interested in the direct numerical simulation of the compressible Euler and Navier Stokes equations at low Mach number regime. First, we propose a review of existing work on the subject in order to identify the issues raised by the simulation of in this kind of flow, and the existing relevant solutions. Then, we will test different selected compressible low Mach solvers using the discontinuous Galerkin space discretisation and discuss about their behaviour.*

Keywords: Compressible flow; Discontinuous Galerkin method; High order; Low Mach number

1 INTRODUCTION

Many flow configurations of practical interest such as jet engine combustion chambers are characterised by the simultaneous presence of a low Mach and acoustic waves. Thus, the capability of accurately simulating this type of systems is of crucial importance when designing the new generation of engines. Tackling this issue numerically is quite a challenging problem. Indeed, it is not only necessary to preserve the acoustic character of the Euler or Navier Stokes equations, which means keeping the compressible property of the flow but also to handle properly the asymptotic behaviour of the equations as the Mach number tends to zero.

Analysing the continuous system by performing double scale asymptotic expansion in power of the Mach number reveals that the limit of the compressible model when the Mach number tends to zero is the incompressible model [8]. However, numerical simulation using classical compressible solvers give solutions which do not converge to the solution of the incompressible system when the Mach number tends to zero [1, 2, 3, 4, 5, 7, 8, 9, 10]. These studies reveal different problems such as the large disparity between the material velocity and the acoustic wave speed which induces stiffness [10], or problems in the scaling of the numerical dissipation introduced by upwind finite volumes low Mach number schemes [4].

In the present work, we propose to perform computations using the discontinuous Galerkin space discretisation and to observe its behaviour for both steady and unsteady low Mach flows.

2 REVIEW OF KNOWN PROBLEMS

2.1 Stiffness problem

The large disparity of wave speeds at low Mach number between flow and acoustic phenomena makes explicit time discretisation inefficient due to very small CFL condition as shown by Turkel [10]. Dealing with this stiffness, this author proposed to precondition the system in order to accelerate the convergence of the solution. Semi-implicit time discretisation as SIMPLE type algorithms are other solutions to cure this problem by implicitly solving the stiff part of the system [3, 7, 10].

2.2 Accuracy problem

As far as stability is concerned, it is well known that upwind schemes introduce viscosity. This numerical dissipation which depends on the Mach number was studied by Guillard and Viozat [4] for the Roe solver. They noticed by performing an asymptotic analysis of both the continuous and discrete systems that the scaling of the numerical dissipation in the Roe solver and the scaling of dissipation in the continuous case were different. This explains the accuracy problem for this solver when the Mach number becomes small. A preconditioning of the flux was proposed for overcoming this problem. Dellacherie [2] extended this analysis to all Godunov type schemes and proposed a cure to all of them.
The study of the unsteady case are rarer, we can cite the recent study by Moguen et al. [7]. In the unsteady case, the pressure-velocity coupling coefficient is of different scale order than that of the steady case. By performing numerical test comparisons these authors show that this coefficient must explicitly depend on the time step.

2.3 Geometry of cells and high order

Feistauer and Kucera [3] performed computations using the discontinuous Galerkin method for space discretisation. Their tests seem to prove the accuracy of their method. However, Rieper and Bader [9] and Guillard [5] have proved that the convergence problem at low Mach number disappear on triangular grids. As the tests carried out in [3] were realised on triangular meshes, accuracy of this method in general geometry can therefore be questioned.
Bassi et al. [1] experimentally confirm the property of triangular meshes with discontinuous Galerkin method. Tests on quadrangular grids are also performed using preconditioned time derivative term and flux function. This method significantly improves the convergence of the solution with polynomial approximations of degrees 1 to 3.

PROPOSED APPROACH

The work proposed here, is to test the behaviour of discontinuous Galerkin methods on compressible low Mach number flow, using the AEROSOL library developed at INRIA [6]. We will particularly focus on the issues presented above by testing the effectiveness the different solutions proposed, like preconditioning methods. Then, we also propose to test the behaviour of higher order methods which are known to be less diffusive.

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